

# Strong 2 – Domination in Fuzzy Graph

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**ABSTRACT:** In this paper we introduce the concepts of strong 2 – domination of a fuzzy graph. We determine the strong 2 – domination number  $\gamma_{2S}$  of the fuzzy graph  $G$  is the minimum cardinality taken over all Strong 2 – dominating number of a fuzzy graph and that of its complement are discussed. We also prove some results on strong 2 – dominating set.

**KEYWORDS:** fuzzy graph, complement, Strong domination, 2 – domination, strong 2 – domination set, strong 2 – domination number.

## I. INTRODUCTION

The study of fuzzy graph and several fuzzy graph analogs of graph theoretic concepts such as paths, cycle and connectedness were started by Rosenfeld [8] (1975). Ore and Berge [1,7] introduced the dominating sets in graphs. Fink introduced the n-domination in graphs. A.Somasundaram and S.Somasundaram [9,10] discussed domination in fuzzy graphs using effective edges in fuzzy graphs. Nagoor Gani and Chandrasekaran discussed domination in fuzzy graphs. They defined the domination using strong arcs [3]. Nagoor Gani and prasanna discussed the concept of an edge domination and edge independence in fuzzy graphs [4,6]. The concept of strong arcs introduced by Bhutini and Rosenfeld [2]. A study of dominating sets in graphs started purely as a problem in the game of chess, it was during 1850s.

## II. PRELIMINARIES

Let us see the following basic definitions in a fuzzy graph and domination in fuzzy graph. Let introduce some new notations

**Definition 2.1:** Let  $E$  is a collection of all two- element subsets of  $V$ . A fuzzy graph  $G = \langle \sigma, \mu \rangle$  is a set with two function  $\sigma: V \rightarrow [0,1]$  and  $\mu: E \rightarrow [0,1]$  such that  $\mu(x,y) \leq \sigma(x) \wedge \sigma(y)$  for all  $x, y \in V$ . hereafter we write  $\mu(x,y)$  or  $\mu(xy)$ .

**Definition 2.2:** A fuzzy graph  $H = \langle \tau, \rho \rangle$  is called a fuzzy subgraph of  $G$  if  $\tau(v_i) \leq \sigma(v_i)$  for all  $v_i \in V$  and  $\rho(v_i, v_j) \leq \mu(v_i, v_j)$  for all  $v_i, v_j \in V$ .

**Definition 2.3:** The order  $p$  and size  $q$  of a fuzzy graph  $G = \langle \sigma, \mu \rangle$  are defined to be  $p = \sum_{x \in V} \sigma(x)$  and  $q = \sum_{xy \in E} \mu(xy)$ .

**Definition 2.4:** The Underlying Crisp Graph of a fuzzy graph  $G = (\sigma, \mu)$  is denoted by  $G^* = (\sigma^*, \mu^*)$ , where  $\sigma^* = \{u \in V / \sigma(u) > 0\}$  and  $\mu^* = \{(u, v) \in V \times V / \mu(u, v) > 0\}$ .

**Definition 2.5:** A fuzzy graph  $G$  is said to be a Strong Fuzzy Graph if  $\mu(x,y) = \sigma(x) \wedge \sigma(y) \forall (x,y)$  in  $\mu^*$  and if Complete Fuzzy Graph if  $\mu(\{x,y\}) = \sigma(x) \wedge \sigma(y) \forall (x,y)$  in  $\sigma^*$ .

# International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

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Vol. 7, Issue 2, February 2018

**Definition 2.6:** Let  $G = (\sigma, \mu)$  be a fuzzy graph on  $V$  and  $S$  subset of  $V$ . Then the *scalar cardinality* of  $S$  is defined to be  $\sum_{v \in S} \sigma(v)$  and it is denoted by  $|S|$ . Let  $p$  denotes the scalar cardinality of  $V$ , also called the order of  $G$ .

**Definition 2.7:** The *Complement* of a fuzzy graph  $G$  denoted by  $\bar{G}$  is defined to be  $\bar{G} = (\sigma, \bar{\mu})$  where  $\bar{\mu}(xy) = \sigma(x) \wedge \sigma(y) - \mu(xy)$ .

**Definition 2.8:** An arc  $(u, v)$  of a fuzzy graph is called an *effective arc* if  $\mu(x, y) = \sigma(x) \wedge \sigma(y)$ . Then  $x$  and  $y$  are called effective neighbors. The set of all effective neighbors of  $u$  is called effective neighborhood of  $x$  is denoted by  $EN(x)$ .

**Definition 2.9:** Let  $G = (\sigma, \gamma)$  be a fuzzy graph on  $V$ . Let  $x, y \in V$ . We say that  $x$  *dominates*  $y$  in  $G$  if  $\mu(x, y) = \sigma(x) \wedge \sigma(y)$ . A subset  $S$  of  $V$  is called a *dominating set* in  $G$  if for every  $v \notin S$ , there exists  $u \in S$  such that  $u$  dominates  $v$ . The minimum fuzzy cardinality of a dominating set in  $G$  is called the *domination number* of  $G$  and is denoted by  $\gamma(G)$  or  $\gamma$ .

**Definition 2.10:** A set  $D$  of nodes of  $G$  is a *strong domination set* of  $G$  if every node  $V(G) - D$  is a strong neighbor of some node in  $D$ .

**Definition 2.11:** The weight of a strong dominating set  $D$  is defined as  $W(D) = \sum_{u \in D} \mu(x, y)$ , where  $\mu(x, y)$  is the membership values (weight) of the strong arcs incident on  $u$ . The *strong domination number* of a fuzzy graph  $G$  is defined as the minimum weight of strong dominating sets of  $G$  and it is denoted by  $\gamma_S(G)$  or  $\gamma_S$ .

**Definition 2.12:** A subset  $D$  of  $V$  is called a *2-domination set* on  $G$  if for every node  $v \in V - D$  there exist atleast two strong neighbors in  $D$ .

**Definition 2.13:** The *2-domination number* of a fuzzy graph  $G$  denoted by  $\gamma_2(G)$ , is the minimum cardinality of a 2-dominating set of  $G$ .

### III. STRONG 2 – DOMINATION IN FUZZY GRAPHS

In this section, we defined strong 2-domination set and strong 2-domination number of a fuzzy graph with suitable examples.

**Definition 3.1:** A set  $D$  of nodes of  $G$  is a strong dominating set and subset  $D$  of  $V$  is called 2-dominating set of  $G$  if for every node  $v \in V(G) - D$  is a two strong neighbor of some node in  $D$  is called *Strong 2-domination in fuzzy graph*.

**Example 3.2:**

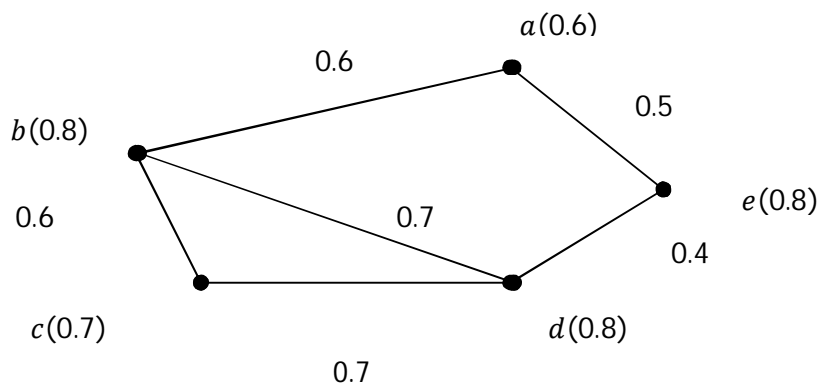


Fig. 1

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Vol. 7, Issue 2, February 2018

**Definition 3.3:** The Strong 2 – domination number of a fuzzy graph  $G$  is defined as the minimum cardinality of a Strong 2 – dominating set of  $G$ . The strong 2 – domination is denoted by  $\gamma_{2S}(G)$  or  $\gamma_{2S}$

**Example 3.4:**

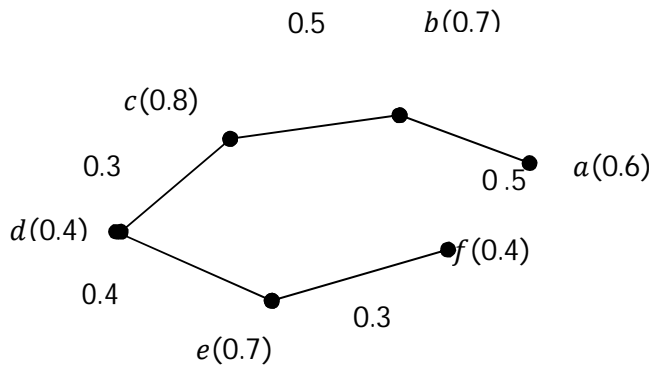


Fig. 2

$\{b, e\}$  is strong 2 – dominating set of a fuzzy graph  $G$ .  $\gamma_{2S}(G) = 2$ .

**Theorem 3.5:** For any fuzzy graph  $G: (V, \sigma, \mu)$ ,  $\gamma_{2S} + \overline{\gamma_{2S}} < 2p$  where  $\overline{\gamma_{2S}}$  is the strong 2 – domination number of  $\overline{G}$  and equality holds if and only if  $0 < \mu(x, y) < \sigma(x) \wedge \sigma(y)$  for all  $x, y \in V$ .

**Proof:** Since  $\gamma_{2S} < p$  and  $\overline{\gamma_{2S}} < p$  ( $p$  is a scalar cardinality). If and only if  $0 < \mu(x, y) < \sigma(x) \wedge \sigma(y)$  and  $0 < \overline{\mu}(x, y) < \sigma(x) \wedge \sigma(y) \Rightarrow \mu(x, y) > 0$ ,

$\gamma_{2S} + \overline{\gamma_{2S}} < 2p$  if and only if  $0 < \mu(x, y) < \sigma(x) \wedge \sigma(y)$ .

**Theorem 3.6[3]:** Every non-trivial connected fuzzy graph  $G$  has a strong 2 – dominating set  $D$  whose complement  $V - D$  is also a strong 2 – dominating set.

**Proof:** Let  $G$  be a strong 2 – dominating graph. Strong 2 – dominating set of  $G$  is  $\{e, c\}$  .....(1). Let  $\overline{G}$  be a strong 2 – dominating graph. Strong 2 – dominating set of  $\overline{G}$  is  $\{e, c\}$  .....(2)

From (1) and (2) Every non-trivial connected fuzzy graph  $G$  has a strong 2 – dominating set  $D$  whose complement  $V - D$  is also a strong 2 – dominating set..Hence the proof.

**Theorem 3.7[3]:** Let  $G$  be a fuzzy graph without isolated nodes. If  $D$  is a minimal strong 2 – dominating set, then  $V - D$  is a strong 2 – dominating set

**Proof:** Let  $d$  be any vertex in  $D$ . Since  $G$  has no isolated vertices, there is a vertex  $c \in N(d)$ . there is a vertex  $c \in V - D$ . Thus, every element of  $D$  is strong 2 – dominated by element  $V - D$ .

**Theorem 3.8:** For any fuzzy graph  $G: (V, \sigma, \mu)$  without isolated nodes  $\gamma_{2S} \leq \frac{p}{2}$

**Proof:** Let  $D$  be a minimal strong 2-domination set of  $G$ . Then  $V - D$  is a Strong 2 – dominating set of  $G$ . therefore  $\gamma_{2S} + \gamma_{2S} \leq p$ ,  $2\gamma_{2S} \leq p \Rightarrow \gamma_{2S} \leq \frac{p}{2}$ .

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(A High Impact Factor, Monthly, Peer Reviewed Journal)

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Vol. 7, Issue 2, February 2018

**Remark 3.1:** let  $G$  be a fuzzy graph such that both  $G$  and  $\overline{G}$  have no isolated nodes, then  $\gamma_{2S} + \overline{\gamma_{2S}} \leq p$  further equality holds if and only if  $\gamma_{2S} = \overline{\gamma_{2S}} = \frac{p}{2}$

**Theorem 3.9:** Every Strong 2 – dominating set of a fuzzy graph  $G$  is a domination set of  $G$ .

**Proof :** Let  $D$  be a strong 2 – dominating set of the fuzzy graph  $G$ . then every node  $V - D$  has two strong neighbours in  $D$ . That is for every node  $v \in V - D$ , there exist minimum two nodes in  $D$  and both dominates  $v$ . Every node in  $V - D$  is dominated by strong two nodes in  $D$ . Thus  $D$  is a dominating set of  $G$ .

**Remark 3.2:** If  $G$  is a fuzzy graph then  $\gamma_{2S}(G) > \gamma(G)$ .

## IV.CONCLUSION

The concept of domination in graph in very rich both in theoretical developments and applications. In this paper the concept of strong 2-domination has been modified for fuzzy graphs .we defined strong 2 –domination set and strong 2 – domination number in fuzzy graphs. Further we proved the theorems based on strong 2 – domination of fuzzy graph.

## ACKNOWLEDGMENTS

Authors would like to thank referees for their helpful comments.

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